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Chiral anomalies and Poincaré invariance*

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Abstract

I consider theories of Dirac fermions with different regularizations, leading to different chiral anomalies. I have investigated the Poincaré invariance properties of these theories. I find that both Lorentz and translational invariance are anomalously violated in general, but that they are respected in a theory that is regularized to give a Wess-Zumino consistent anomaly (the Bardeen anomaly). In a theory that is regularized to give a covariant anomaly, Poincaré invariance is not respected, and in this case I calculate the divergence of the angular momentum current and energy-momentum tensor in a nonabelian theory with external vector and axial vector sources.

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1 Introduction

Chiral anomalies have many important applications in particle physics [1]. For example, they are relevant for the decay $\pi^0 \to \gamma\gamma$, for the solution to the $U(1)$ puzzle, and for finding constraints on the fermion content in gauge theories. Anomalies appear due to the regularization of the fermionic part of the theory, and represents a quantum mechanical breakdown of the classical symmetries of the fermion. When such breakdown occurs, the currents associated with these symmetries are not conserved, but have the anomalies as their divergence.

There are certain ambiguities connected with the anomalies. If we consider for example the theory of a nonabelian fermion coupled to external vector (V) and axial vector (A) sources, it is known that it is possible to regularize the theory so that the divergence of the vector current vanishes and the divergence of the axial vector current is equal to the Wess-Zumino consistent chiral anomaly [2]. Another possibility is to have covariant anomalies [3], in which case both the divergence of the current and of the axial current are different from zero.

In this paper I consider the problem of the Poincaré invariance properties of these differently regularized theories. To my knowledge this has not been investigated before. I have found that Poincaré invariance is not respected for many regularizations – rather, it is broken by anomalies. For example, the theory with covariant regularization is not Poincaré invariant. I calculate the

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anomalous divergence of the angular momentum current and energy-momentum tensor in this case. On the other hand, if the theory is regularized to give the Bardeen anomaly, which is consistent, then the angular momentum current and energy-momentum tensor have vanishing divergences, and Poincaré invariance is intact.

It may be worth pointing out that the Poincaré anomalies in this paper are not directly related to the well known gravitational anomalies [4]. In the gravitational case, the anomalies appear from Feynman diagrams with a number of energy-momentum tensors at the vertices. In the Poincaré case on the other hand, they appear from diagrams with one energy-momentum tensor and a number of vectors and axial vectors (for the translational anomalies), or one angular momentum current and a number of vectors and axial vectors (for the Lorentz anomalies).

The organization of the paper is the following. In sec. 2, I discuss the nonabelian "*VA*-theory", and in particular the global symmetries of this model. In sec. 3, I discuss the regularization of the quantum theory of the *VA*-theory. The regularization scheme I use is proper time regularization in Minkowski space, where it is seen that the exact specification of the regularization is controlled by a "regularization operator" $\tilde{D}$, related to the Dirac operator D. I then discuss various choices for $\tilde{D}$. In sec. 4, I show that the choice that leads to the Wess–Zumino consistent Bardeen anomaly leads to Lorentz and translational invariance. On the other hand, in sec. 5, I show that the choice that leads to the covariant anomaly is not Poincaré invariant, and I calculate the divergence of the angular momentum current and the energy-momentum current. Sec. 6 is a brief conclusion.

2 The *VA*-theory

The nonabelian *VA*-theory is a fairly standard theory of a Dirac fermion, and suitable for illustration of the main points. It is given by the Lagrangian

$$\mathcal{L} \;=\; \bar{\psi}(D+i\epsilon)\psi, \qquad D \;=\; i\partial\!\!\!/ - V\!\!\!\!/ - A\!\!\!/\gamma_5 - \mu. \tag{1}$$

Here, $V_\mu = V_\mu^a t^a$ and $A_\mu = A_\mu^a t^a$ are external vector and axial vector sources, respectively, in the Lie algebra of some group with generators t^a. I have added a small mass μ as an infrared regulator, as well as an $i\epsilon$. I will usually suppress both of these.

Let us consider the global phase rotations

$$\psi \;\to\; e^{i\alpha}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{-i\alpha}, \qquad \alpha \;=\; \alpha^a t^a, \tag{2}$$

and chiral rotations

$$\psi \;\to\; e^{i\beta\gamma_5}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{i\beta\gamma_5}, \qquad \beta \;=\; \beta^a t^a. \tag{3}$$

It is convenient to assign the transformation rules

$$V_\mu \;\to\; e^{i\alpha}V_\mu e^{-i\alpha}, \qquad A_\mu \to e^{i\alpha}A_\mu e^{-i\alpha}, \tag{4}$$

and

$$V_\mu \;\to\; e^{-i\beta\gamma_5}V_\mu e^{-i\beta\gamma_5}, \qquad A_\mu \to e^{-i\beta\gamma_5}A_\mu e^{-i\beta\gamma_5}. \tag{5}$$

to the external sources since this makes the Lagrangian invariant. From these transformation rules we can derive the naive conservation equations

$$\begin{aligned} \partial_\mu J^{a\mu} &= 0, \qquad J_\mu^a \equiv \bar{\psi}\gamma_\mu t^a\psi, \\ \partial_\mu J_5^{a\mu} &= 0, \qquad J_\mu^{5a} \equiv \bar{\psi}\gamma_\mu\gamma_5 t^a\psi, \end{aligned} \tag{6}$$

where J_μ and J^5_μ are the fermion current and axial current. (If V_μ and A_μ transformed trivially under the symmetry, we would find that these currents were "covariantly" conserved instead of "ordinarily" conserved.) The fact that the symmetry under consideration is global rather than local means that the anomalies we will discuss in the following sections are "symmetry anomalies" rather than "gauge anomalies".

Translations with the parameter a_μ act on the fields by

$$
\begin{aligned}
\psi &\rightarrow e^{ia_\mu P^\mu}\psi, \\
\bar\psi &\rightarrow \bar\psi e^{-ia_\mu P^\mu}, \\
V_\mu &\rightarrow e^{ia_\nu P^\nu}V_\mu e^{-ia_\nu P^\nu}, \\
A_\mu &\rightarrow e^{ia_\nu P^\nu}A_\mu e^{-ia_\nu P^\nu},
\end{aligned} \tag{7}
$$

where $P_\mu \equiv i\partial_\mu$ are the generators, and Lorentz transformations with parameters $\omega_{\mu\nu}$ act by

$$
\begin{aligned}
\psi &\rightarrow e^{i\frac{1}{2}\omega_{\mu\nu}J^{\mu\nu}}\psi, \\
\bar\psi &\rightarrow \bar\psi e^{-i\frac{1}{2}\omega_{\mu\nu}J^{\mu\nu}},
\end{aligned} \tag{8}
$$

with generators $J_{\mu\nu} \equiv \frac{1}{2}\sigma_{\mu\nu} + (ix_\mu\partial_\nu - x_\nu i\partial_\mu) \equiv S_{\mu\nu} + L_{\mu\nu}$ and

$$
\begin{aligned}
V_\mu &\rightarrow (e^{i\frac{1}{2}\omega_{\rho\sigma}[J^{\rho\sigma}]})_\mu{}^\nu V_\nu, \\
A_\mu &\rightarrow (e^{i\frac{1}{2}\omega_{\rho\sigma}[J^{\rho\sigma}]})_\mu{}^\nu A_\nu,
\end{aligned} \tag{9}
$$

with generators $[J^{\rho\sigma}]_{\mu\nu} \equiv i(g^\rho{}_\mu g^\sigma{}_\nu - g^\sigma{}_\mu g^\rho{}_\nu) + (x^\rho i\partial^\sigma - x^\sigma i\partial^\rho)g_{\mu\nu}$. This leads to the conservation of the energy-momentum tensor $\Theta^{\mu\nu}$,

$$
\partial_\mu\Theta^{\mu\nu} = 0, \qquad \Theta^{\mu\nu} \equiv \bar\psi\gamma^\mu i\partial^\nu\psi, \tag{10}
$$

and the angular momentum current $J^{\mu,\alpha\beta}$,

$$
\partial_\mu J^{\mu,\alpha\beta} = 0, \qquad J^{\mu,\alpha\beta} \equiv \bar\psi\left[\tfrac{1}{2}\{\gamma^\mu, \tfrac{1}{2}\sigma^{\alpha\beta}\} + \gamma^\mu L^{\alpha\beta}\right]\psi, \tag{11}
$$

on the classical level.

3 Quantization

For the sake of the discussion, it is useful to recall some basics about the quantization of a theory such as the VA-theory. The quantum theory is given by the path integral

$$
Z = \int \mathcal{D}\psi\mathcal{D}\bar\psi e^{i\int d^4x\bar\psi D\psi} \equiv \mathrm{Det}D, \tag{12}
$$

which also defines the fermionic determinant. The effective action W ($Z \equiv e^{iW}$) is given by

$$
W = -i\mathrm{Tr}\ln D. \tag{13}
$$

These expressions are formal and will become well defined by regularization.

For infinitesimal parameters, the rotations of the fermions induce a change in the Dirac operator

$$
\begin{aligned}
D &\rightarrow e^{-i\alpha+i\beta\gamma_5}De^{i\alpha+i\beta\gamma_5} \\
&= D + i(D\alpha - \alpha D) + i(D\beta\gamma_5 + \beta\gamma_5 D) \\
&\equiv D + \delta D,
\end{aligned} \tag{14}
$$

which in turn induces a change in W:

$$\delta W = -i \mathrm{Tr} \delta D \frac{1}{D}. \tag{15}$$

The Jacobian J is then determined by the requirement that the path integral Z is unchanged under a change of variables:

$$Z = J e^{iW + i\delta W} = e^{iW}. \tag{16}$$

Defining now a Lagrangian by $J \equiv \exp(i \int d^4 x \mathcal{L}_J)$, we have

$$\int d^4 x \mathcal{L}_J = -\delta W = i \mathrm{Tr} \delta D \frac{1}{D}. \tag{17}$$

$\mathcal{L}_J$ thus contains the anomalies, G^a_α and G^a_β,

$$\mathcal{L}_J \equiv \alpha^a G^a_\alpha + \beta^a G^a_\beta, \tag{18}$$

and is therefore the quantity of interest.

When a regularization is specified, both the transformations (2) and (3) are in general seen to be broken symmetries. By the Noether procedure, we can find the conservation equations for the currents:

$$\begin{aligned} \partial_\mu J^{a\mu} &= -G^a_\alpha, \\ \partial_\mu J^{a\mu}_5 &= -G^a_\beta. \end{aligned} \tag{19}$$

Thus, if the anomalies are nonvanishing, the symmetries are violated.

The main point of this paper is that a similar thing may happen to the Poincaré transformations. Indeed, following the Noether procedure, we must consider local variations

$$\delta D = i(D \tfrac{1}{2}\{a_\mu, P^\mu\} - \tfrac{1}{2}\{a_\mu, P^\mu\} D) \tag{20}$$

for the translations, and

$$\delta D = i(D \tfrac{1}{2}\{\tfrac{1}{2}\omega_{\mu\nu}, J^{\mu\nu}\} - \tfrac{1}{2}\{\tfrac{1}{2}\omega_{\mu\nu}, J^{\mu\nu}\} D) \tag{21}$$

for the Lorentz transformations. Here I have used the symmetrized products

$$\begin{aligned} a_\mu P^\mu &\to \tfrac{1}{2}\{a_\mu, P^\mu\} = \tfrac{1}{2} a_\mu P^\mu + \tfrac{1}{2} P_\mu a^\mu, \\ \tfrac{1}{2}\omega_{\mu\nu} J^{\mu\nu} &\to \tfrac{1}{2}\{\tfrac{1}{2}\omega_{\mu\nu}, J^{\mu\nu}\} = \tfrac{1}{4}\omega_{\mu\nu} J^{\mu\nu} + \tfrac{1}{4} J_{\mu\nu}\omega^{\mu\nu}, \end{aligned} \tag{22}$$

which secures hermiticity when a_μ and $\omega_{\mu\nu}$ are local functions. As I will show, the Jacobian may be nontrivial for this case as well,

$$\mathcal{L}_J = a_\mu G^\mu_a + \tfrac{1}{2}\omega_{\mu\nu} G^{\mu\nu}_\omega, \tag{23}$$

leading to the anomalous divergence equations

$$\partial_\mu J^{\mu,\alpha\beta} = -G^{\alpha\beta}_\omega \tag{24}$$

for the angular momentum current, and

$$\partial_\mu \Theta^{\mu\nu} = -G^\nu_a \tag{25}$$

for the energy-momentum tensor. Regularizations where $G^\omega_{\mu\nu}$ and G^a_μ are nonvanishing therefore violates Poincaré symmetry.

4 Proper time regularization

The regularization scheme I have used for my calculations is proper time regularization in Minkowski space [5]. Traditionally proper time regularization is used in connection with the Euclidean formalism, see e.g. the review [6]. However, it turns out to be a great advantage to work in Minkowski space, when different regularizations – consistent, covariant, etc. – are discussed, since in that case the various prescriptions are conveniently controlled by the choice of a "regularization operator" $\tilde{D}$, related to the Dirac operator D. This will be discussed in the next sections; see also [5]. There is also no need to perform analytical continuations of the fields and transformations in Minkowski space. But other regularization schemes should of course be possible.

I introduce a proper time integral and the operator $\tilde{D}$ in the following way:

$$\begin{aligned}\int d^4x\mathcal{L}_J &= i\mathrm{Tr}\delta D\frac{1}{D} \\ &= i\mathrm{Tr}\delta D\tilde{D}\frac{1}{D\tilde{D}} \\ &= \int_{1/\Lambda^2}^{\infty} ds\mathrm{Tr}\delta D\tilde{D}e^{is(D\tilde{D}+i\epsilon)}\end{aligned} \tag{26}$$

The operator $\tilde{D}$ is a priori arbitrary, except that it must be chosen to give the right ϵ-prescription. This will then ensure convergence at the upper integration limit. For the lower integration limit the cutoff Λ is introduced, which is to be taken to infinity at the end of the calculation. When the appropriate choice for $\tilde{D}$ is made, $\mathcal{L}_J$ will be regular and well defined.

We can use the expression for δD from eq. (14) to perform the proper time integral and write $\int d^4x\mathcal{L}_J$ in a Fujikawa-like form [7, 8]:

$$\begin{aligned}\int d^4x\mathcal{L}_J &= \int_{1/\Lambda^2}^{\infty} ds\mathrm{Tr}\, i(D\alpha - \alpha D + D\beta\gamma_5 + \beta\gamma_5 D)\tilde{D}e^{isD\tilde{D}} \\ &= -\mathrm{Tr}\,\alpha\left(e^{i\tilde{D}D/\Lambda^2} - e^{iD\tilde{D}/\Lambda^2}\right) - \mathrm{Tr}\,\beta\gamma_5\left(e^{i\tilde{D}D/\Lambda^2} + e^{iD\tilde{D}/\Lambda^2}\right).\end{aligned} \tag{27}$$

Here I have used the cyclicity of the trace, the identity $\tilde{D}e^{isD\tilde{D}}D = \tilde{D}De^{is\tilde{D}D}$, and the fact that only the lower limit of the integral contributes due to the implicit presence of the ϵ. Similarly, for the Lorentz transformations and translations we have

$$\begin{aligned}\int d^4x\mathcal{L}_J &= -\mathrm{Tr}\,\tfrac{1}{2}\{\tfrac{1}{2}\omega_{\mu\nu}, J^{\mu\nu}\}\left(e^{i\tilde{D}D/\Lambda^2} - e^{iD\tilde{D}/\Lambda^2}\right) \\ &\quad -\mathrm{Tr}\,\tfrac{1}{2}\{a_\mu, P^\mu\}\left(e^{i\tilde{D}D/\Lambda^2} - e^{iD\tilde{D}/\Lambda^2}\right).\end{aligned} \tag{28}$$

To proceed from here it is necessary to choose a $\tilde{D}$.

5 Consistent regularization

Let us make the choice

$$\tilde{D} = (i\gamma_5)D(i\gamma_5). \tag{29}$$

Using the cyclicity of the trace and the fact that γ_5 commutes with α, we have

$$\int d^4x\mathcal{L}_J = -2\mathrm{Tr}\,\beta\gamma_5 e^{i\tilde{D}D/\Lambda^2}. \tag{30}$$

The terms proportional to α in eq. (27) have thus canceled out.

We can find out which kind of chiral anomaly this choice of $\tilde{D}$ leads to. After a standard calculation we get (see e.g. [9])

$$\begin{aligned}\mathcal{L}_J \;=\; \frac{1}{4\pi^2}\mathrm{tr}\beta\,\Big[&\epsilon_{\mu\nu\rho\sigma}\big(\tfrac{1}{4}F_V^{\mu\nu}F_V^{\rho\sigma}+\tfrac{1}{12}F_A^{\mu\nu}F_A^{\rho\sigma}\\ &\qquad -\tfrac{2}{3}iA^\mu A^\nu F_V^{\rho\sigma}-\tfrac{2}{3}iA^\mu F_V^{\nu\rho}A^\sigma-\tfrac{2}{3}iF_V^{\mu\nu}A^\rho A^\sigma\\ &\qquad -\tfrac{8}{3}A^\mu A^\nu A^\rho A^\sigma\big)\\ &-\tfrac{2}{3}\{D_\mu^V A_\nu,\{A^\mu,A^\nu\}\}+\tfrac{1}{3}\{D_\mu^V A^\mu,A^2\}+\tfrac{2}{3}i[D_\mu^V F_V^{\mu\nu},A_\nu]\\ &-\tfrac{1}{6}[F_{\mu\nu}^A,F_V^{\mu\nu}]+\tfrac{1}{3}D_V^2 D_\mu^V A^\mu-2A_\mu D_\nu^V A^\nu A^\mu\Big]\end{aligned} \tag{31}$$

Here $F_V^{\mu\nu}$ and $F_A^{\mu\nu}$ are the Bardeen tensors,

$$\begin{aligned}F_{\mu\nu}^V &\equiv \partial_\mu V_\nu-\partial_\nu V_\mu+i[V_\mu,V_\nu]+i[A_\mu,A_\nu],\\ F_{\mu\nu}^A &\equiv \partial_\mu A_\nu-\partial_\nu A_\mu+i[V_\mu,A_\nu]+i[A_\mu,V_\nu],\end{aligned} \tag{32}$$

and $D_\mu^V \equiv \partial_\mu + i[V_\mu,\,\cdot\,]$.

The terms proportional to the ϵ-tensor is the familiar Bardeen anomaly [10]. The other terms are of even intrinsic parity [11]. (Note, incidentally, that it is not possible to remove these terms by a redefinition of the path integral since we are explicitly considering a global symmetry. Only in the case where the chiral transformations are gauge transformations can we remove these terms by adding a suitable polynomial in V_μ, A_μ, and derivatives, to the Lagrangian. This is because the transformation rules (4) and (5) should then be replaced with gauge transformations, where in particular A_μ picks up a gradient.) The Bardeen anomaly is known to be consistent [2], hence the regularization resulting from $\tilde{D}$ is a consistent regularization.

Actually, this is not the whole story: One also gets a term proportional to Λ^2 (and terms proportional to $1/\Lambda^2$, $1/\Lambda^4$,...) in addition to the Λ-independent terms in eq. (31). It is necessary to somehow remove this term if we intend to take Λ to infinity at the end of the calculation. This can be done by the Pauli–Villars inspired regularization described in [9]; see also [5]. This implies a "theorem" that we can simply drop all Λ-dependent terms.

It is easy to see that not only the terms proportional to α in eq. (27) cancel out, but also the terms proportional to a_μ and $\omega_{\mu\nu}$ in eq. (28). Hence the Lorentz and translational anomalies – $G_{\mu\nu}^\omega$ and G_μ^a – vanish for this regularization.

6 Covariant regularization

Let us now instead make the choice

$$\tilde{D} \;=\; D^c \;\equiv\; -CD^TC^{-1}. \tag{33}$$

This is the "charge conjugate" of D; C is the charge conjugation matrix and transposition is with respect to the Dirac matrix structure.

It is now no longer true that terms proportional to α in eq. (27) automatically cancel. For a combination of a phase rotation with parameter $\alpha = \alpha^a t^a$ and chiral phase rotation with parameter $\beta = \beta^a t^a$ we get

$$\begin{aligned}\mathcal{L}_J \;=\; \frac{1}{16\pi^2}\,\epsilon_{\mu\nu\rho\sigma}\mathrm{tr}\,\Big[&\alpha(F_V^{\mu\nu}F_A^{\rho\sigma}+F_A^{\mu\nu}F_V^{\rho\sigma})\\ &+\beta(F_V^{\mu\nu}F_V^{\rho\sigma}+F_A^{\mu\nu}F_A^{\rho\sigma})\Big]\,.\end{aligned} \tag{34}$$

This is the expression for the covariant anomaly [6].

The presence of a term proportional to α means that the current is not conserved. This is well known for a theory that is regularized to have a covariant anomaly [6, 1].

This form of $\tilde{D}$ is similar to that of $D^\dagger$ in the Euclidean formulation, where also the sign of $\not{A}\gamma_5$ is changed relative to $i\not{\partial} - \not{V}$. In the Euclidean case the operator $D^\dagger$ has a special status, since it is used for the construction of the positive operators $DD^\dagger$ and $D^\dagger D$. The positivity of these operators will then ensure the convergence of the upper limit of the proper time integral, instead of the $i\epsilon$ which has the same effect in Minkowski space. For this reason the Euclidean formalism produces "naturally" the covariant anomaly, and it takes a considerable amount of work to produce other anomalies, such as the consistent one [6].

Within this regularization, we can now calculate the Jacobian that corresponds to the translations and Lorentz transformations. The procedure, which also includes some nonstandard elements, is essentially the one described in ref. [5], where it was used in a slightly different context. The result is

$$\begin{aligned}\mathcal{L}_\omega &= \frac{1}{8\pi^2}\omega_{\mu\nu}\mathrm{tr}\Big[\tfrac{1}{6}\Box\tilde{F}_A^{\mu\nu} + \tfrac{1}{6}i\partial_\rho[V^\rho,\tilde{F}_A^{\mu\nu}] + \tfrac{1}{6}i\partial_\rho[A^\rho,\tilde{F}_V^{\mu\nu}] \\ &\qquad + x^\mu(2V^\nu F_V\tilde{F}_A + A^\nu F_V\tilde{F}_V)\Big] \\ &\equiv \tfrac{1}{2}\omega_{\mu\nu}G_\omega^{\mu\nu} \end{aligned} \tag{35}$$

and

$$\begin{aligned}\mathcal{L}_a &= \frac{1}{8\pi^2}a_\mu\mathrm{tr}(2V^\mu F_V\tilde{F}_A + A^\mu F_V\tilde{F}_V) \\ &\equiv a_\mu G_a^\mu. \end{aligned} \tag{36}$$

Thus, as advertized, the angular momentum current and energy-momentum tensor are not conserved, and Poincaré symmetry is violated.

7 Conclusion

We have seen that in the nonabelian VA-theory considered in this paper, Poincaré symmetry survives quantization for a regularization that leads to the consistent Bardeen form of the chiral anomaly, while it is anomalously broken for a regularization that leads to the covariant chiral anomaly. Of course many other choices for the regularization operator $\tilde{D}$ can be made. (Indeed many other choices than proper time regularization can be made for the regularization scheme itself.) In general Poincaré symmetry is violated since eq. (28) does not vanish for general choices for $\tilde{D}$. Thus, one lesson to be learned from the calculations in this paper is that it is necessary to check whether or not Poincaré symmetry may be anomalously broken when a given regularization is adopted.

It would be interesting to try to generalize the investigation of Poincaré symmetry to theories of chiral fermions. It would also be interesting to investigate if, and how, Poincaré anomalies may cancel in gauge theories where the fermion content ensures a cancellation of the chiral anomalies.

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